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COHERENT STRUCTURE-REFLECTIVE TURBULENT VISCOUS FLOW

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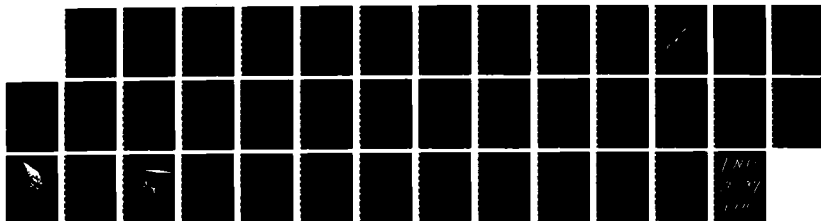
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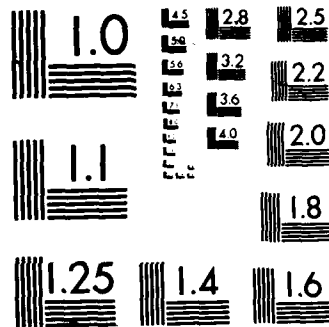
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ANNUAL REPORT

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COHERENT STRUCTURE-REFLECTIVE TURBULENT VISCOUS FLOW MODELING

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| <p>Utilizing a multiple-element scale/coherence decomposition of the Navier-Stokes equations, the essential characteristics of the large scale turbulent structure are computed in wall-bounded shear flows. The effect of small-scale turbulence structure is modeled and the large-scale turbulence structure is computed assuming weakly non-linear large-scale dynamics. The effects of large-scale non-linearity and the presence of wave-like elements in the flow are accounted for utilizing perturbation theory. The resultant propagation, evolution (in the convected reference frame) and (statistical) occurrence of three-dimensional vortical instabilities are computed and compared to experimental data. Subsequently, coherent structure reflective turbulence models shall be constructed from this analysis.</p> | | | |
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Executive Summary

Utilizing a multiple-scale decomposition of the Navier-Stokes equations, it has been demonstrated that non-equilibrium fine-scale stress modeling is sufficient to generate linearly unstable large scale vortical disturbances. By assembling various of these modes, the (short-time) characteristics of coherent structure may be simulated and, eventually solved via an initial value problem.

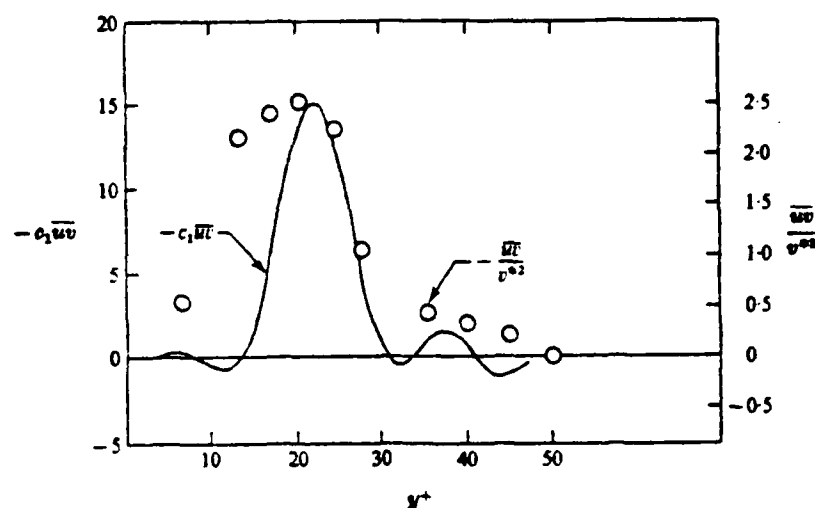
Discussion

The present research program seeks to make fundamental contributions to the understanding of wall-bounded turbulent shear flows in constant and (spatially/temporally) variable pressure conditions. Both the advection and (Lagrangian) evolution in the advected frame of reference of large scale coherent structures are focused upon. As a consequence, basic deterministic vortical dynamics results as well as modeling implications are sought. In connection with the modeling, the deterministic simulations will eventually be ensembled utilizing a pdf representation of the burst occurrence, along with a temporal chaos assumption at some 'upstream' location.

This work has a foothold in both the relatively distant past and current state-of-the-art analysis/simulation methodologies. With respect to the historical references, the research relies heavily on the following developments:

- I. Liepmann's 'turbular' fluid concept, and
- II. Landahl's wave-guide model of turbulence.

If a thumbnail synopsis of the work was required it would be "to investigate the merging of I and II". That is, to extend Landahl's weakly non-linear wave-like model of large-scale turbulence by accounting for the complex apparent constitution of the medium induced by the small scale processes. That such an approach might be successful is supported by comparisons between the measured and predicted Reynolds stresses from Orr-Sommerfeld computations, presented by Kim, Kline and Reynolds.



Comparison of the form of the Reynolds stress measured from H_2 bubble data during a bursting period with calculations using two-dimensional steady linearized equations (Orr-Sommerfeld). Scales are arbitrary. \circ , Burst period data, $t = 0-6.7$ sec; —, two-dimensional lines theory.

The modern analysis foothold of the research involves Large Eddy Simulation (LES), in which the filtered equations of motion are used to resolve deterministically the largest scales of motion while the unresolved sub-grid scale (SGS) processes are modeled. These SGS processes are the rigorous theoretical equivalent of Liepmann's 'turbular' fluid. Unfortunately, although LES shows promise of being managed and maturing into a reliable predictive tool, it is often an un-controlled combination of interacting numerical/filtering/modeling difficulties and cannot yet be assumed to provide stand-alone 'truth' data. In fact, even direct numerical simulations (DNS), which resolve all scales of motion, are regularly beset with confusion regarding the true solution in low Reynolds number turbulent flows.

For this reason, the research sought to isolate the essence of LES, while avoiding some of the problems presently associated with it. The multiple-scale nature of the flow has been confronted explicitly yet idealized by employing the discrete/disparate scale dependent variable decomposition of Mollo-Christensen and (for non-steady flows) the coherent-incoherent element decomposition popularized by Hussain & Reynolds and also used by J.T.C. Liu. Utilizing the merged (Mollo-Christensen/Hussain & Reynolds) decomposition (see Attachment 1.), the interactions and transitions between:

- Large-scale temporally coherent,
- Large-scale temporally incoherent, and
- Small-scale temporally incoherent

elements may be focused upon.* Clearly, in this formulation the large-scale temporally incoherent element is the experimentally-observed (spatially) coherent structure.

As a result of this approach to implementing the wave-guide model for a 'turbular' fluid, most of the numerical and filtering problems of LES may be eliminated, while addressing the small-scale process modeling, as it affects the large-scale vortical dynamics. To guide this modeling, the full moment equation formulation of the unresolved processes developed by Deardorff has been used. In so doing, current algebraic/equilibrium/isotropic approximations may be shown to fail in certain situations. The work is, therefore, focusing on the need for non-equilibrium and anisotropic unresolved process models, relative to the prediction of large-scale turbulent structures.

*The utility of (compounded) signal-background decompositions has also been evaluated but, due to various mathematical difficulties, has not proven as susceptible to analysis.

In the course of computing weakly non-linear large-scale structure with various small-scale process models, certain specific deterministic vortical events are being examined for their relevance to turbulence. With regard to the physics, the burst propagation and evolution of the structures are the focus of interest. At that point when results are applied to modeling, the statistical occurrence shall be confronted. Interestingly, however, the statistical occurrence probability distribution function equation is generically identical to the (L) propagation equation being studied:*

$$L_t + U_c L_x = PL + U_c C_f$$

where:

$$U_c \equiv U/H$$

$$H \equiv \text{shape factor, and}$$

$$P \equiv \text{generalized pressure gradient.}$$

It has been found that, utilizing U_c , various experimentally-derived propagation speed characterizations may be understood theoretically. Results for a given U_c , when 'unwound' utilizing the resultant H evolution, imply the velocity field which sustains the flow. In addition, the dynamics of the superlayer and interface between the rotational and irrotational flow is dependent upon:

$$U - U_c,$$

*In addition, this is the form of the multiple-scale perturbation amplitude evolution equation associated with the: 1) presence of a coherent non-steady component, 2) effect of residual non-linearities, and 3) effect of large scale incoherent on small scale incoherent structures. These effects are essential to the understanding of turbulence perpetuation.

with entrainment also proportional to the amplitude of the interfacial oscillations. The interaction/synchronization between this outer flow phenomenon and the other flowfield instabilities to be computed is a primary focus of the work. The pressure field is clearly a key diagnostic variable for this phenomenon.

Throughout this work, attention will be focused on DL/Dt , the flow normal to the wall. Traditionally in laminar boundary layer analysis, the vertical velocity 'goes along for the ride'. In turbulent flow, the unsteady vertical velocity as a source of curvature may be a driver, although non-parallel mean flow effects have not yet been shown to be crucial.

Note that, defining P^{-1} as a time constant, τ , this equation may be written:

$$\tau DL/Dt + L = L_{eq.} \quad ,$$

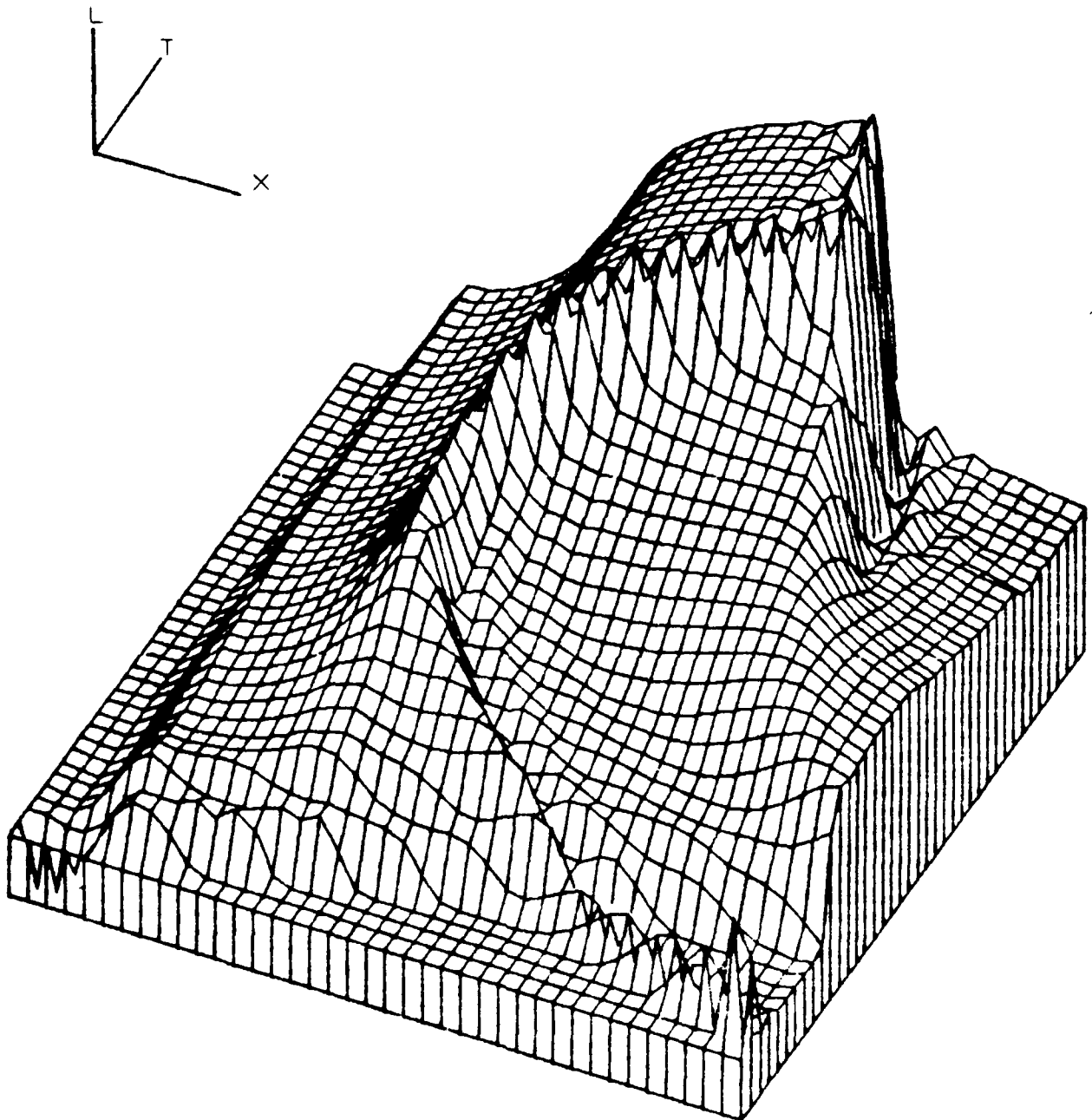
where:

$$D/Dt \equiv \partial/\partial t + U_c \partial/\partial x, \text{ and}$$

$$L_{eq.} \equiv \tau U_c C_f \quad .$$

Note that, in this form, the equation expresses a first order lag between L and its equilibrium value, in advected coordinates. A representative solution is presented here, which illustrates various stages of exponential growth, both spatial and temporal.

TYPICAL L EVOLUTION COMPUTATION

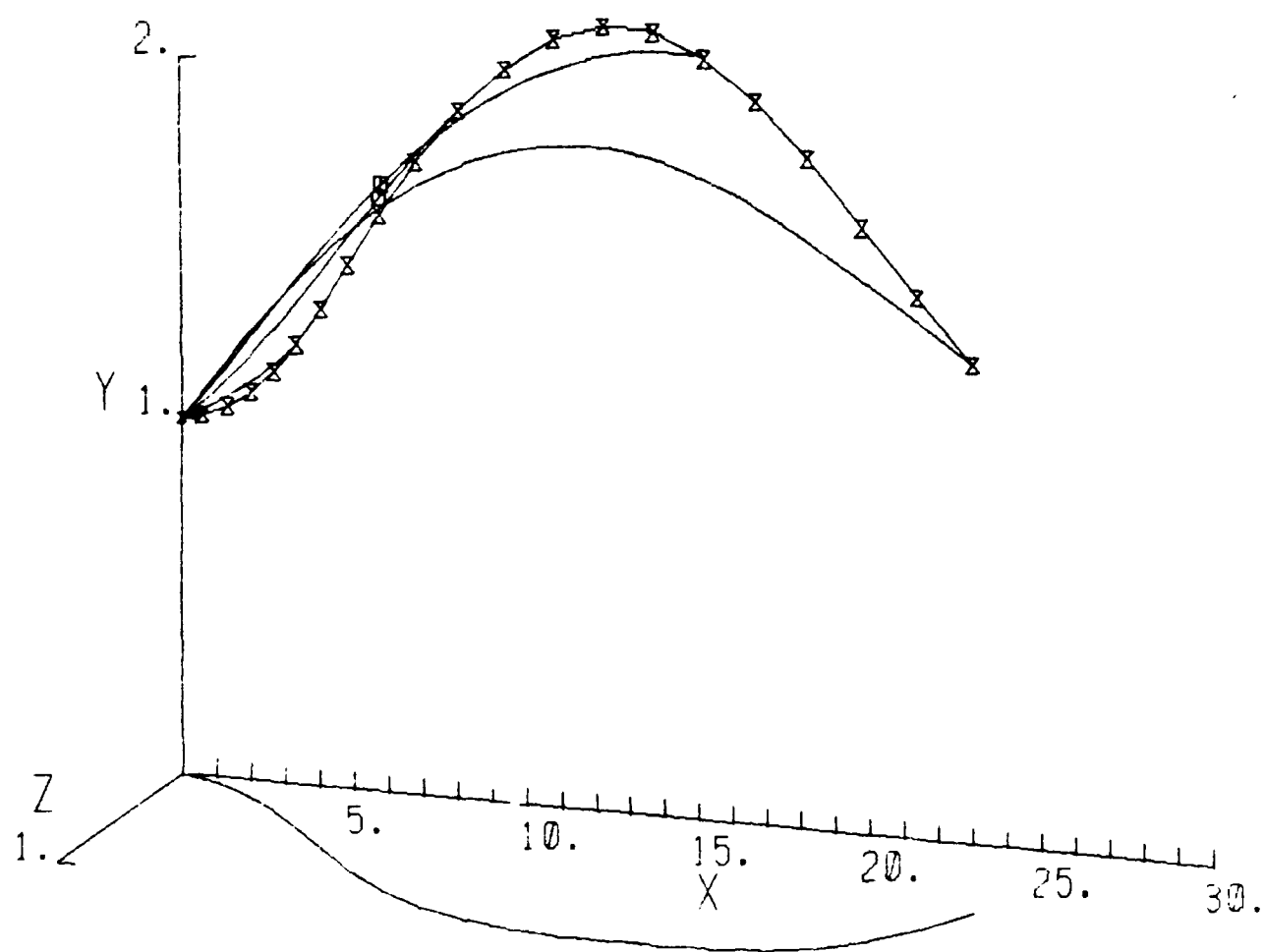


With regard to evolution of the turbulent burst structures, heavy reliance is placed on the flowfield visualizations of Blackwelder; Kline; Willmarth, etc. Inasmuch as these results generally involve streakline patterns, the computational work is presented in that form, also. Unfortunately, the pathline/streakline visualization suffers from two flaws:

1. Given a streakline, a unique inference of the velocity field is usually not possible, and
2. Streaklines can assume a complex appearance in very simple flows (the work of Saric is illuminating here).

In spite of these cautions, the comparison between computations and experiment in this format is a crucial test of the research hypotheses. A sample pathline/streakline plot is presented here, however, extensive use of this tool awaits the computation of the dominant unstable modes of oscillation in the shear flow.

TYPICAL THREE-DIMENSIONAL PATHLINE/STREAKLINE PLOT



Fundamentally, the view of turbulence adopted here is relatively traditional, following along the lines of Willmarth. Specifically, we assume that the low wavenumber pressure field measured at the wall results from the large-scale motions in the outer part of the flow, and that these are triggered by the bursting in the buffer layer, which is caused in turn by secondary instabilities growing on inflectionary profiles induced by longitudinal vortices in the sub- and buffer layers.

In the weakly non-linear formulation which we are pursuing (prior to utilizing DNS/LES tools) several of these physical observations are (or may be) related to classical mathematical features of the Fourier/Laplace transformed normal mode vorticity equations which are being solved. (This dispersion relation/vertical structure distribution calculation exercise is the first step in a general initial value problem computation.)

FOURIER AND LAPLACE TRANSFORMED 3-D VORTICITY EQUATIONS
LINEARIZED LARGE SCALE MOTIONS

$$\left\{ (\bar{u}-c) + \frac{1}{\alpha} [(\text{Re}^{-1} + \epsilon)(D^2 - k^2) + (D^2 \epsilon)] \right\} \hat{\omega}_1 = - \left\{ (D\bar{u}) + \frac{1}{\alpha} (D\epsilon)(2D^2 - k^2) \right\} \hat{u}_3$$

$$- \frac{\beta}{\alpha} \left\{ (D\epsilon)D - (2D^2 \epsilon) \right\} \hat{u}_2$$

$$\left\{ (\bar{u}-c) + \frac{1}{\alpha} [(\text{Re}^{-1} + \epsilon)(D^2 - k^2) + (D\epsilon)D] \right\} \hat{\omega}_2 = - \frac{\beta}{\alpha} (D\bar{u}) \hat{u}_2$$

$$\left\{ (\bar{u}-c) + \frac{1}{\alpha} [(\text{Re}^{-1} + \epsilon)(D^2 - k^2) + (D^2 \epsilon)] \right\} \hat{\omega}_3 = \left\{ (D\bar{u}) + \frac{1}{\alpha} (D\epsilon)(2D^2 - k^2) \right\} \hat{u}_1$$

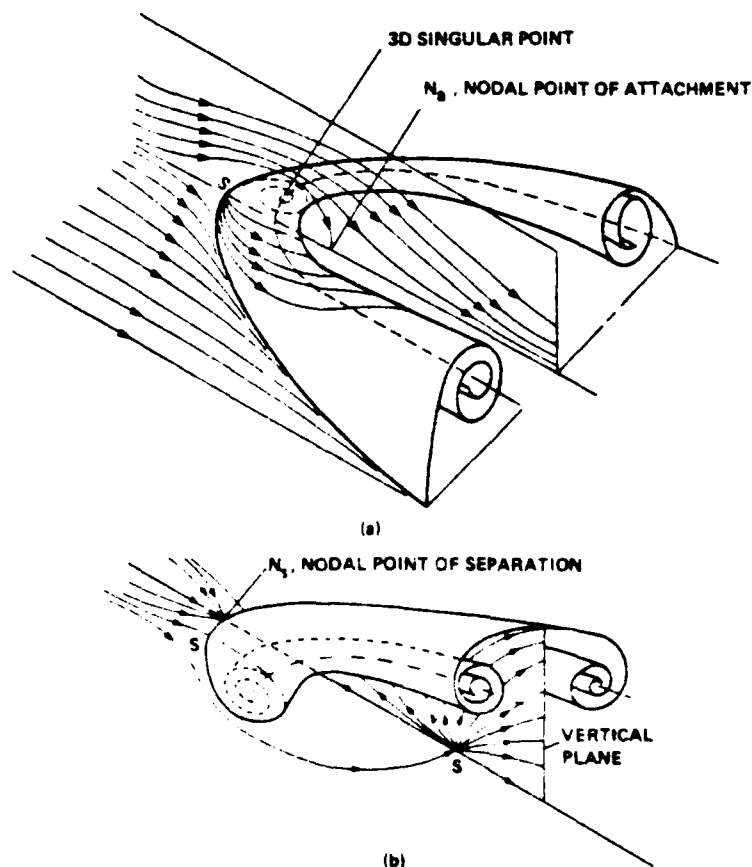
$$+ \left\{ (D\epsilon)D - (2D^2 \epsilon) \right\} \hat{u}_2$$

$$+ \left\{ - \frac{1}{\alpha} [D^2 \bar{u} + (D\bar{u})D] \right\} \hat{u}_2$$

First, the inflectional profile should be computable from the U'_i evolution equations, given the appropriate r'_{ij} model, which reflects sub-layer structures. This brings into question the role of critical level processes, as in laminar flow, and their relationship to the observed free shear layer instability. In addition, the (Benney-Gustavson) vertical velocity-vertical vorticity resonance (or near resonance) may be associated with the generation of large vertical structures which deflect laterally the vertically-sheared flow, thus generating streamwise vorticity*:

$$\left\{ (\bar{U}-c) + \frac{1}{\alpha} [(\text{Re}^{-1} + \epsilon)(D^2 - k^2) + (D\epsilon)D] \right\} \tilde{\omega}_2 = - \frac{\beta}{\alpha} (D\bar{U}) \tilde{u}_2$$

The Klebanoff hairpin vortex and the horseshoe vortex arising in local 3-D unsteady separation/re-attachment may be related to this phenomenon.



Dividing surfaces formed from combinations of (a) nodal point of attachment and saddle point. (b) nodal point of separation and saddle point.

Although not in vogue for many years, the effect of local transient adverse pressure gradients and unsteady separations was espoused by Prandtl to be an essential aspect of the nature of turbulence. The generation of these vortical structures and the presence of favorable and adverse pressure fields also clearly indicates the possibility for encountering classical vortex breakdown phenomena. In its discussion, characterizing the helicity of the flow is also becoming a useful tool.

Finally, work has exposed the role of spatially-variable apparent viscosities on the generation of streamwise vorticity. This effect is reminiscent of the formation of Langmuir vortices and was carried out to correct an error in the literature, as well as to expose the effect of strong vertical variations of viscosity on the generation of 3-D vorticity.

In order to predict such vortical events, or the tendency toward them, the weakly non-linear equations of motion are being solved with various un-resolved process models. The fundamental problems with this aspect of the work involve the non-uniqueness of such modeling and the lack of data on small-scale turbulence, conditionally-sampled to reflect its dependence on the burst events. Therefore, as systematically as possible, the work is tediously surveying the effect of non-equilibrium, anisotropy and non-linearity in the un-resolved process model on the large-scale structure prediction. At this point, due to the importance of the effect on turbulence modeling in general and, also, for visco-elastic media* (see Crow; Davis; Betchov and Criminale), the effect of non-equilibrium is being concentrated upon. Based on a recurrent symmetry of various turbulent process modeling, the first-order lag model has also been adopted here:

*The small-scale turbulence field is known to behave in such fashion also.

$$\tau D r'_{ij} / Dt + r'_{ij} = r'_{ijeq}.$$

Various candidate non-linear and anisotropic 'stress' models are also being analyzed. Unfortunately, the non-uniqueness problem is even more prevalent here. The work is, therefore, attempting to determine whether or not small scale structure non-equilibrium is an essential aspect of the turbulent burst dynamics processes.

The analysis is able to handle arbitrary velocity profiles as a function of space and time. However, relative to TBL structures, the data to validate a large-eddy model is concentrated in simpler flows. Therefore, until the essential features of the un-resolved process modeling are clarified, classical data shall be the focus of simulation. In fact, to supplement the constant pressure TBL simulations which have been on-going, the transitional flow of Schubauer and Klebanoff is being addressed to prove the model in a situation in which a (streamwise) change in instability processes is occurring, as the mean vorticity distribution changes and unresolved process effects arise.

Typical results for an unstable mode of the predicted disturbance vorticity are presented here for a (first-order lag) unresolved process model. Various parametric sweeps through the burst:

1. Frequency,
2. Span-wise wave number,
3. Complex streamwise wave number,
4. Non-equilibrium time-constant, and
5. Unresolved process model constants,

are being carried out for these transitional profiles to isolate the essential ingredients for a small-scale turbulence model which allows the observed large scale structure streaklines and pathlines to be simulated.

The conditions for which this outer flow disturbance was found are:

$$\begin{aligned}\beta &= 0.1 && \text{(Span-wise wavenumber)} \\ \omega &= 2.0 && \text{(Frequency)} \\ \epsilon &= 100 && \text{(Peak small scale stress divided by } \nu) \\ \tau_{\epsilon} &= 1.0 && \text{(Non-equilibrium time-constant)} \\ \text{Re} &= 4.4369 \times 10^4\end{aligned}$$

where all variables are non-dimensionalized by δ^* and U_{∞} . The streamwise wavenumber solution is:

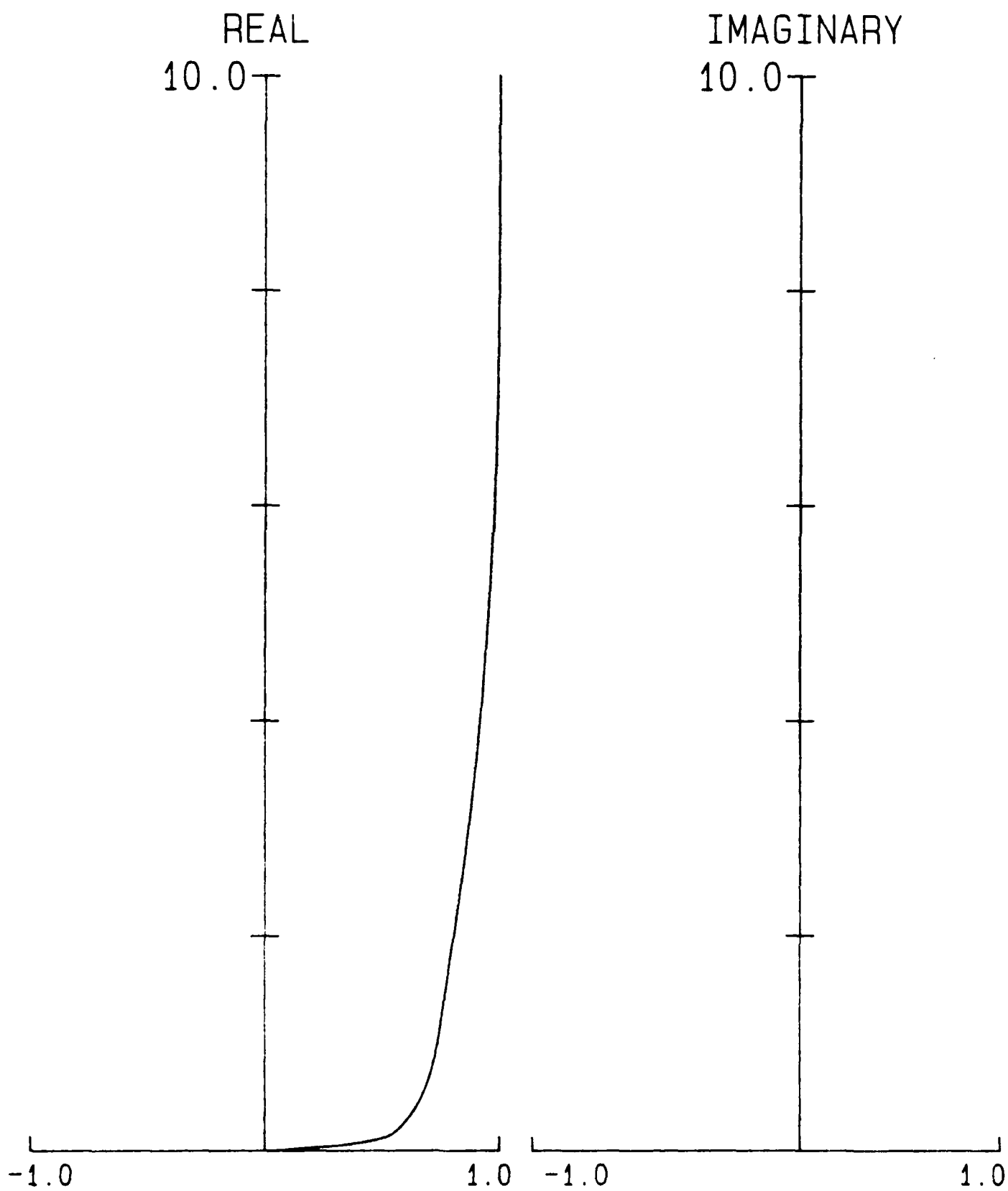
$$\alpha = .162 - 0.037i.$$

Note that the angle of the disturbance in the horizontal plane conforms with experimental observations also. For this case the amplitude distribution of the small-scale stresses was fit to the function $y^2 e^{-Ay^2}$ in order to simulate traditional experimental data such as those in the attachment here.

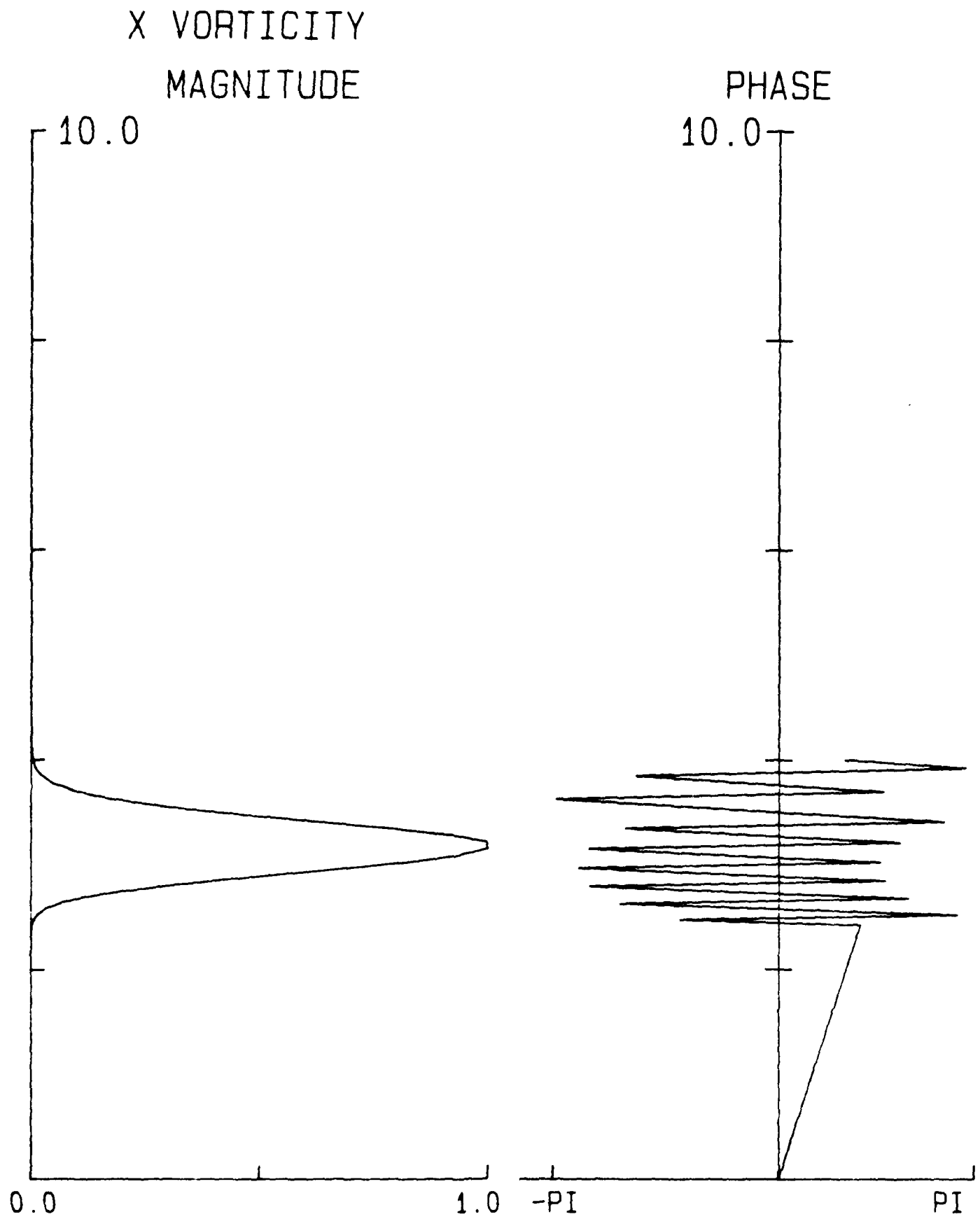
Equally important, however, is the population/character of the solutions. A typical plot of the boundary condition error (wall normal velocity) is shown here. Note that the various error spikes always neighbor zero error points, i.e. solutions. For the disturbance solution presented, the coarse resolution and fine resolution error fields are also presented.

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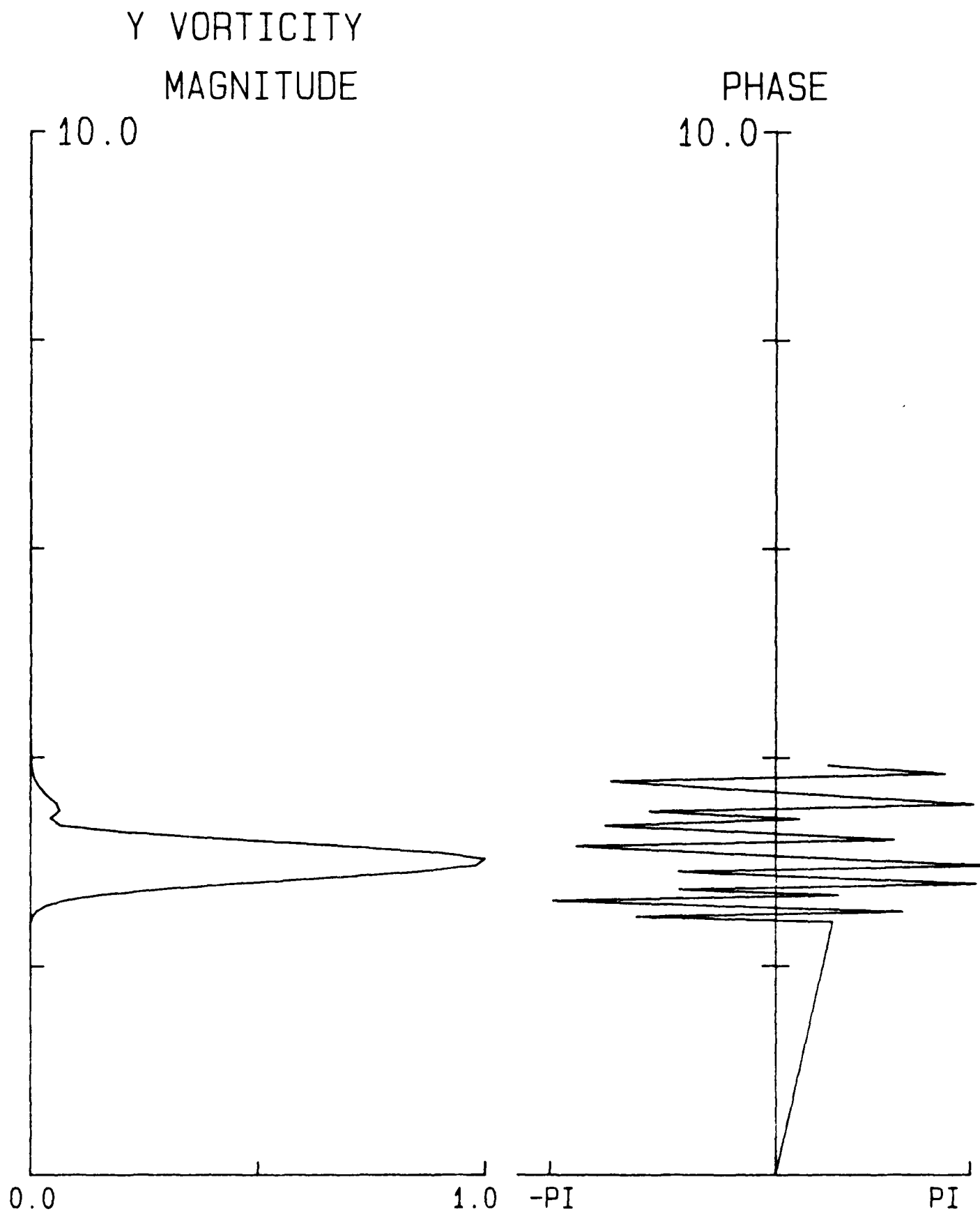
U VELOCITY PROFILE



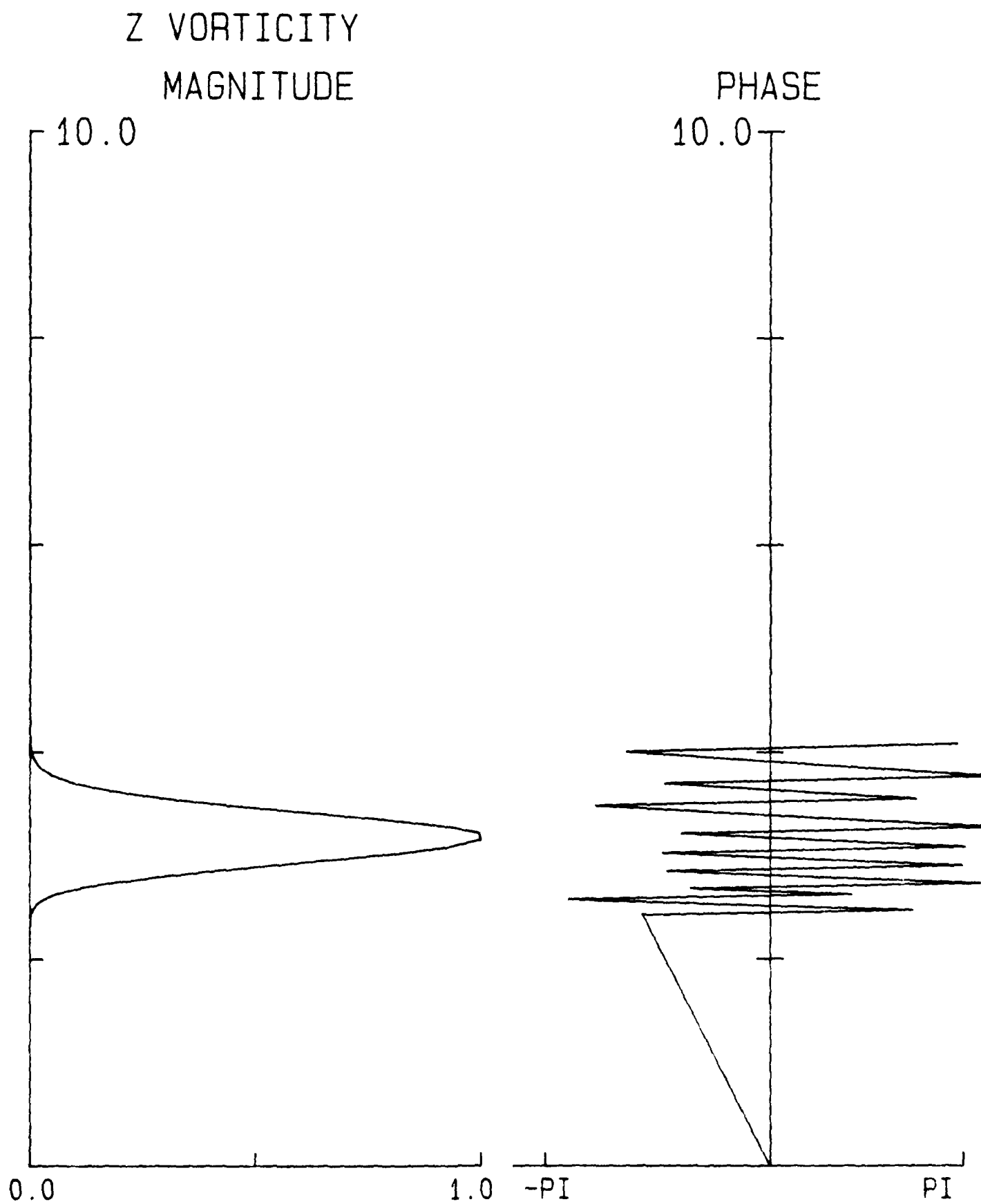
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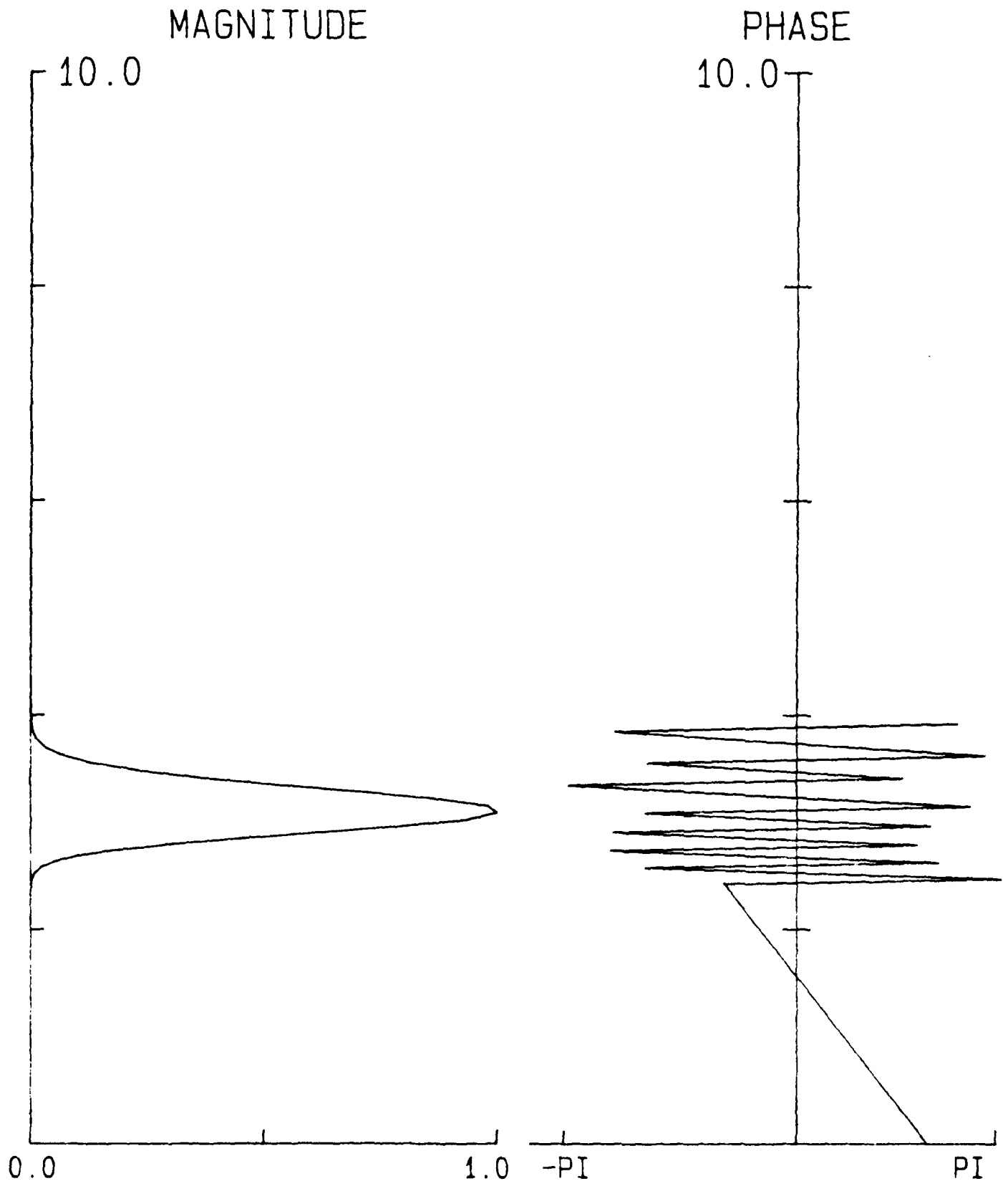


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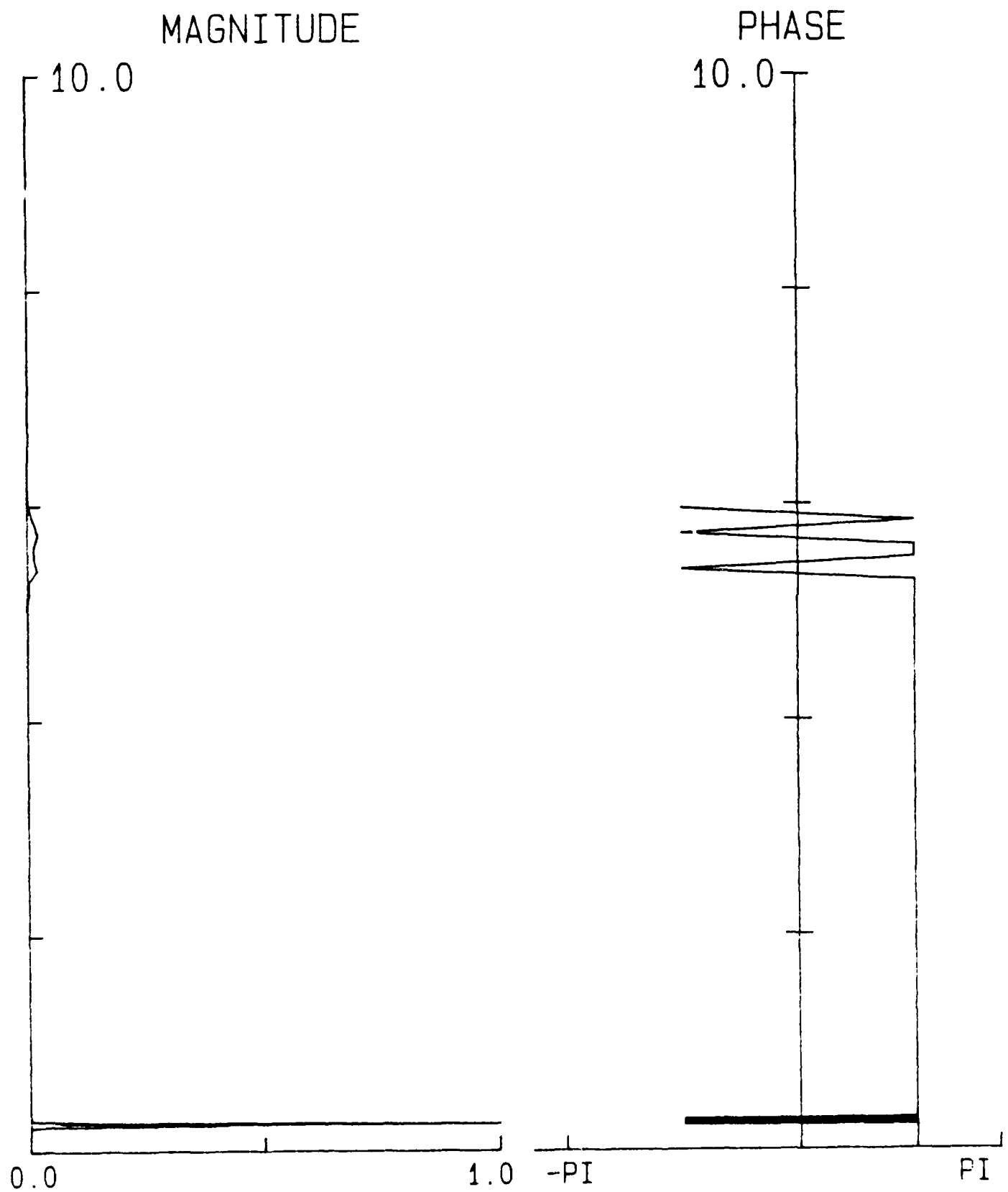
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PRESSURE STRESS PROFILE



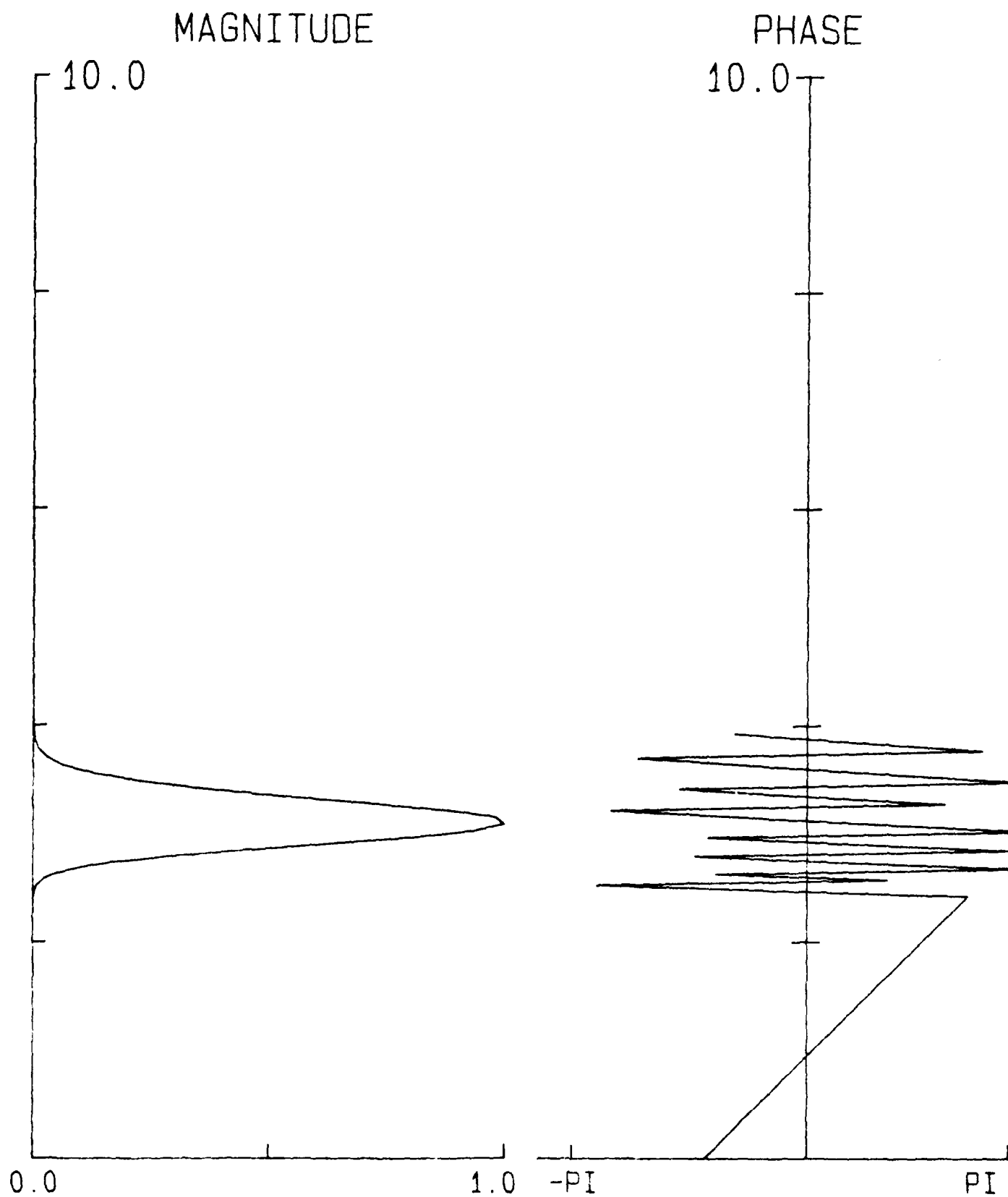
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TXY STRESS PERTUBATION



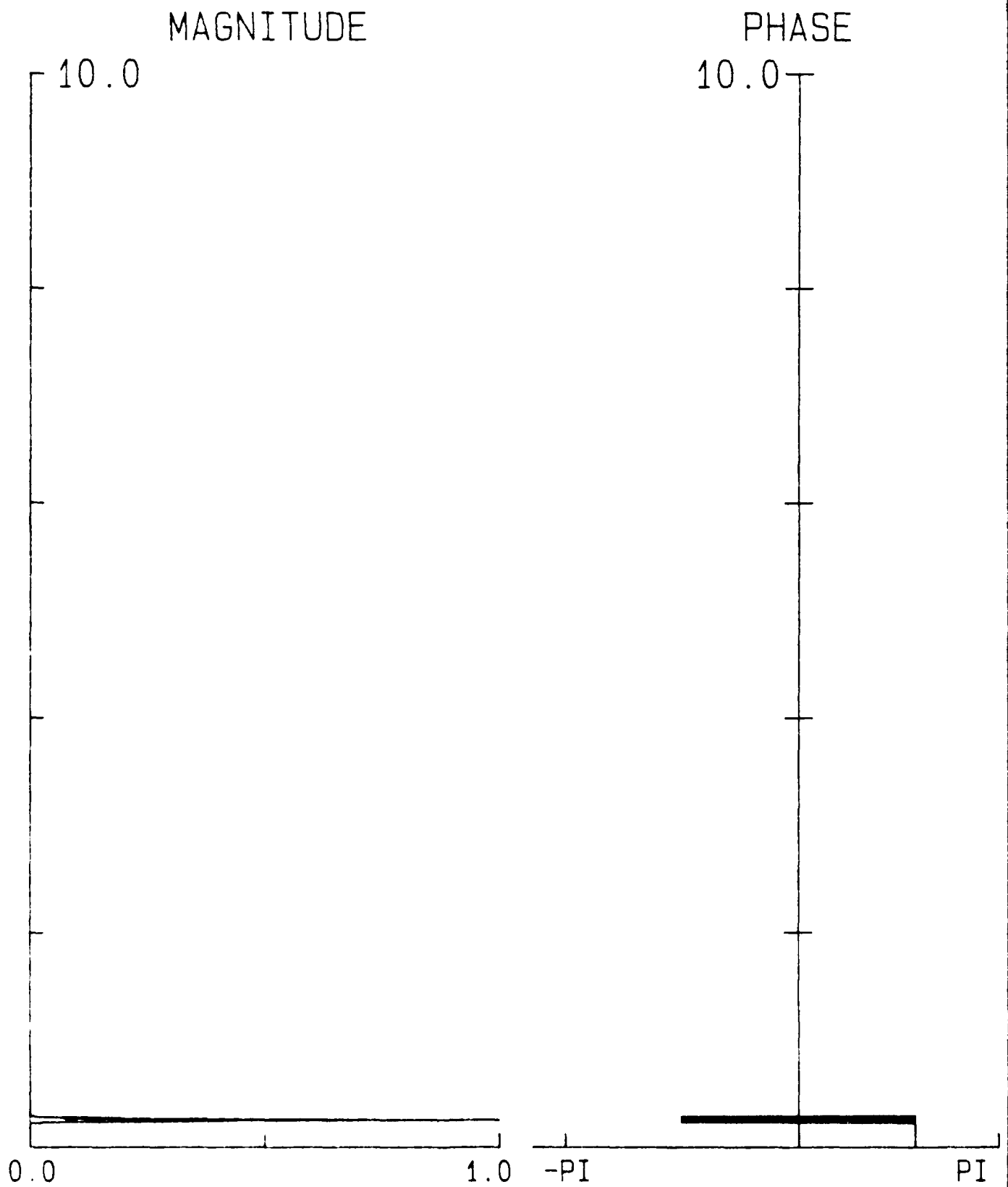
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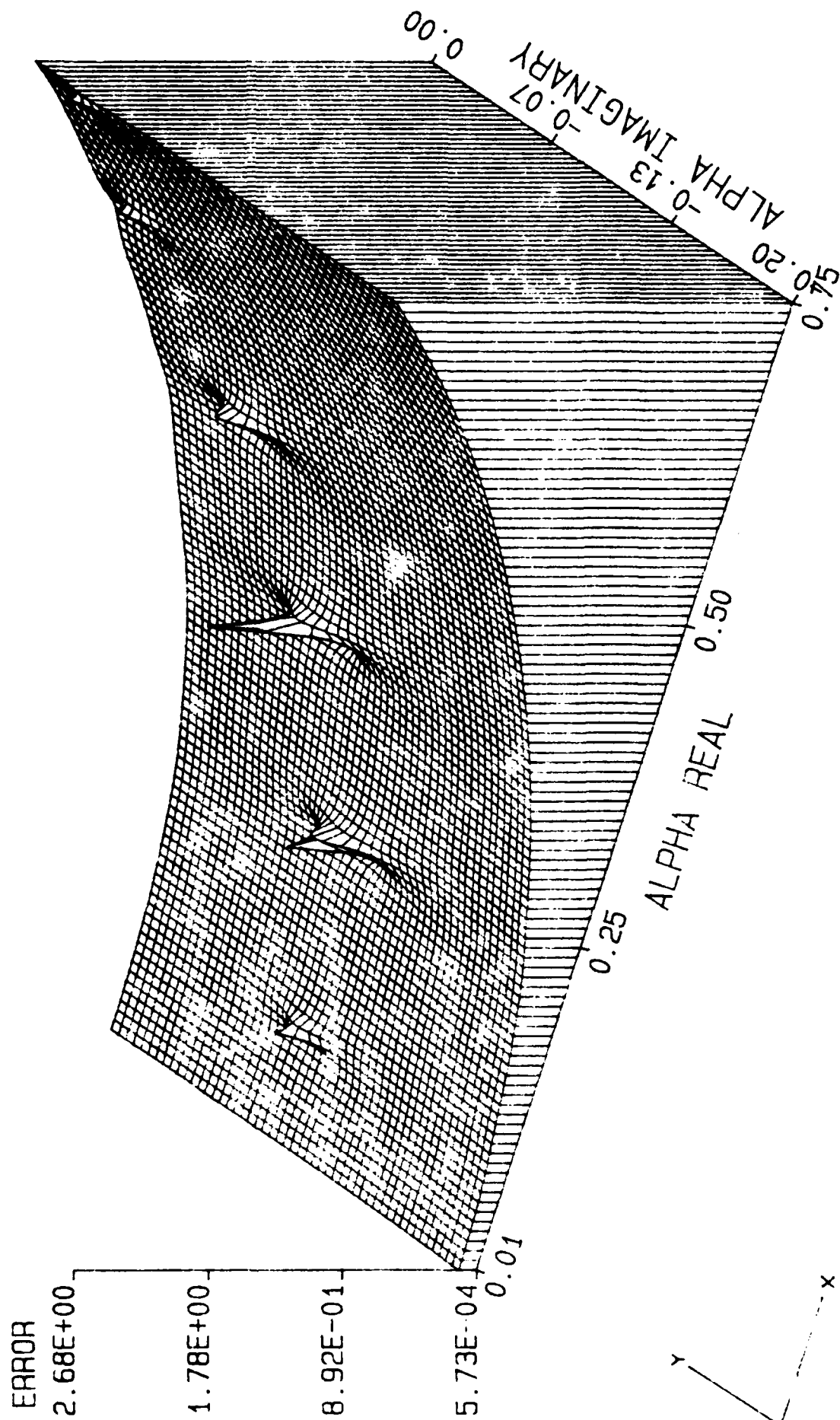
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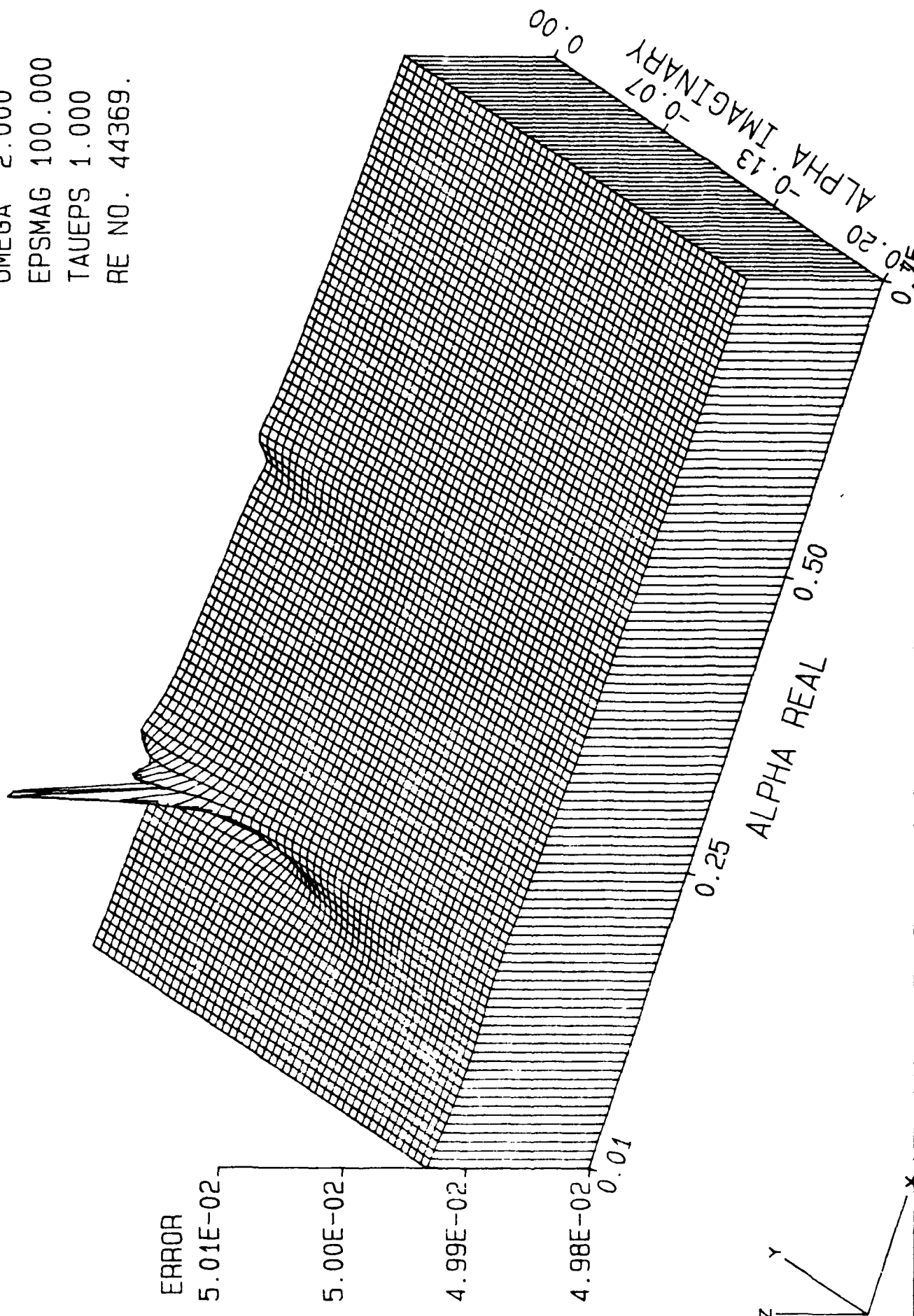
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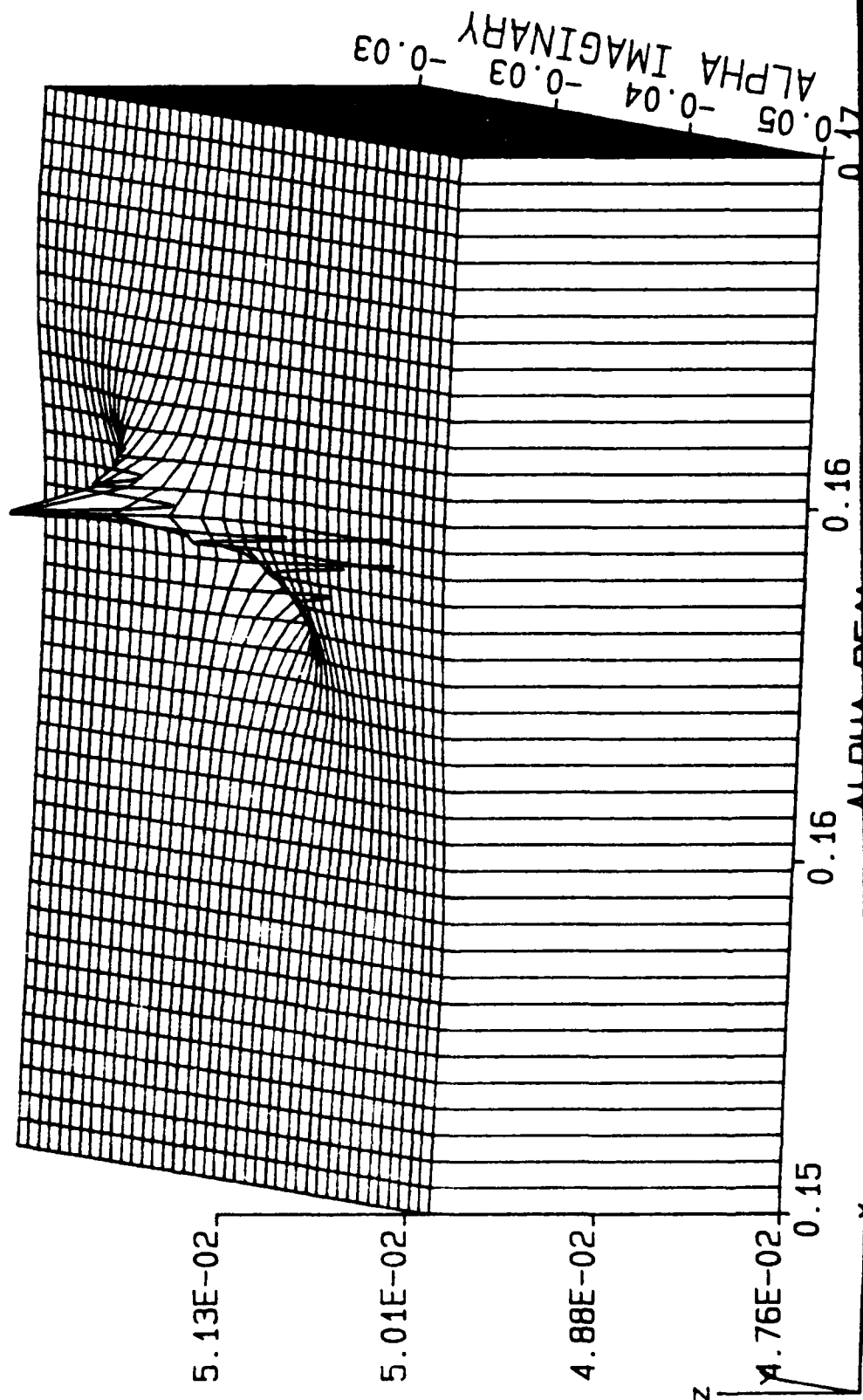
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ERROR



Conclusions

During the forthcoming year of work, additional variable pressure velocity fields shall be addressed as the quality of the unresolved process modeling warrants. In addition, the research hypothesis (that the merging of the wave-guide model of turbulence with an appropriate complex small scale turbulence constitutive model will clarify some of the turbulence vortical dynamics and the modeling thereof) shall be thoroughly tested. If the weakly non-linear modeling is successful, the SGS modeling implications for LES will be significant. If the wave-guide model is not susceptible to "improvement", it will be clear that full non-linearity is required to simulate even the initial stages of the burst process and that the un-resolved process modeling is not critical, as long as it reflects reasonably the scales of motion. In such an event, the wave-guide model would be viewed as a tool to understand the nature of an existing fluctuation field rather than its inception and evolution.

Publications Supported by AFOSR

1. "Vorticity with Variable Viscosity," AIAA J., May 1986.
2. "Turbulent Boundary Layers with Vectored Mass Transfer," AIAA J., March 1986.
3. "Boundary Conditions for Flow Over Permeable Surfaces," JFM, September 1985.
4. "The Use of Angle-Magnitude Variables to Describe Turbulent Shear Flows and the Modeling Thereof," First World Congress on Computational Mechanics, September 1986.
5. "The Effect of Streamwise Velocity Non-Uniformity on Vortex Breakdown in Adverse Pressure Gradients," - to be published.

Bibliography

- Benney, D.J. and Gustavsson, L. Håkan, "A New Mechanism for Linear and Nonlinear Hydrodynamic Instability," *Studies in Applied Mathematics*, Vol. 64, pp. 185-209, 1981.
- Betchov, R., "Stability of Parallel Flows With Frequency - Dependent Viscosity," *Phys. Fl.*, Vol. 8, pp. 1910-1911.
- Blackwelder, R.F., "Streamwise Velocity Profiles During the Bursting Phenomenon," *APS, Division of Fluid Dynamics*, 1972.
- Crow, S.C., "Visco-Elastic Properties of Fine-Grained Incompressible Turbulence," *JFM*, Vol. 33, part 1, pp. 1-20.
- Davis, R.E., "Perturbed Turbulent Flow, Eddy Viscosity and the Generation of Turbulent Stresses," *JFM*, Vol. 63, Part 4, pp. 673-693.
- Deardorff, *Boundary Layer Meteorology*, 1974.
- Kim, H.T., Kline, S.J. and Reynolds, W.C., "The Production of Turbulence Near a Smooth Wall in a Turbulent Boundary Layer," *JFM*, Vol. 50, Part 1, pp. 133-160, 1971.
- Hussain, A.K.M.F. and Reynolds, W.C., "The Mechanics of an Organized Wave in Turbulent Shear Flow," *JFM*, Vol. 41, Part 2, pp. 241-258; *Ibid.* "Part 2. Experimental Results," *JFM*, Vol. 54, Part 2, pp. 241-261; *Ibid.* "Part 3. Theoretical Models and Comparisons With Experiments," *JFM*, Vol. 54, Part 2, pp. 263-288.
- Landahl, M.T., "A Wave-Guide Model For Turbulent Shear Flow," *JFM*, Vol. 29, Part 3, pp. 441-459, 1967. "Wave Mechanics of Breakdown," *JFM*, Vol. 56, Part 4, pp. 775-802, 1972.
- Liepmann, H.W., "Free Turbulent Flows," In The Mechanics of Turbulence, p. 211, *Int. Symp. Nat. Sci. Res. Centre, Marseille*, 1961, *Gordon & Breach*, 1964.
- Liu, J.T.C., "Developing Large - Scale Wavelike Eddies and the Near Jet Noise Field," *JFM*, Vol. 62, Part 3, pp. 437-464, 1974.
- Mollo-Christensen, E., "Physics of Turbulent Flow," *AIAA J.*, Vol. 9, No. 7, pp. 1217-1228, 1971.
- Schubauer, G.B. and Klebanoff, P.S., "Contributions on the Mechanics of Boundary-Layer Transition," *NACA TN 3469*, 1955.
- Willmarth, W.W., "Structure of Turbulence in Boundary Layers," Advances in Applied Mechanics, Vol. 15, pp. 159-254, 1975.

ATTACHMENT 1

A FOUR-ELEMENT DECOMPOSITION OF THE NAVIER-STOKES EQUATIONS

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} \quad (1)$$

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (2)$$

Decompose the flow into four components by:

$$\left. \begin{aligned} U_i &= U_i + \bar{U}_i + U'_i + u'_i \\ P &= P + \bar{P} + P' + p' \end{aligned} \right\} \quad (3)$$

where: U_i, P = time mean flow

\bar{U}_i, \bar{P} = coherent component

U'_i, P' = incoherent component, large scale

u'_i, p' = incoherent component, small scale

Mean Flow:

$$U_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} = \frac{\partial}{\partial x_j} (\overline{U_i U_j} + \bar{R}_{ij} + \bar{r}_{ij})$$

$$\frac{\partial U_i}{\partial x_i} = 0$$

Coherent Flow:

$$\frac{\partial \bar{U}_i}{\partial t} + U_j \frac{\partial \bar{U}_i}{\partial x_j} + \bar{U}_j \frac{\partial U_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{U}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} (\bar{R}_{ij} + \bar{r}_{ij})$$

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0$$

Incoherent Large Scale Flow:

$$\frac{\partial U'_i}{\partial t} + (U_j + \bar{U}_j) \frac{\partial U'_i}{\partial x_j} + U'_j \frac{\partial}{\partial x_j} (U_i + \bar{U}_i) = - \frac{1}{\rho} \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 U'_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \{r'_{ij}\}$$

$$\frac{\partial U'_i}{\partial x_i} = 0$$

Incoherent Small Scale Flow

$$\frac{\partial u'_i}{\partial t} + (U_j + \bar{U}_j + U'_j) \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial}{\partial x_j} (U_i + \bar{U}_i + U'_i) = - \frac{1}{\rho} \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_j \partial x_j}$$

$$\frac{\partial u'_i}{\partial x_i} = 0$$

where $\overline{\quad}$ = time average

$\langle \quad \rangle$ = phase average

$\{ \quad \}$ = large scale average

Define fluid stresses:

$$R_{ij} \equiv U_i' U_j' , \quad \text{and}$$

$$r_{ij} \equiv u_i' u_j' .$$

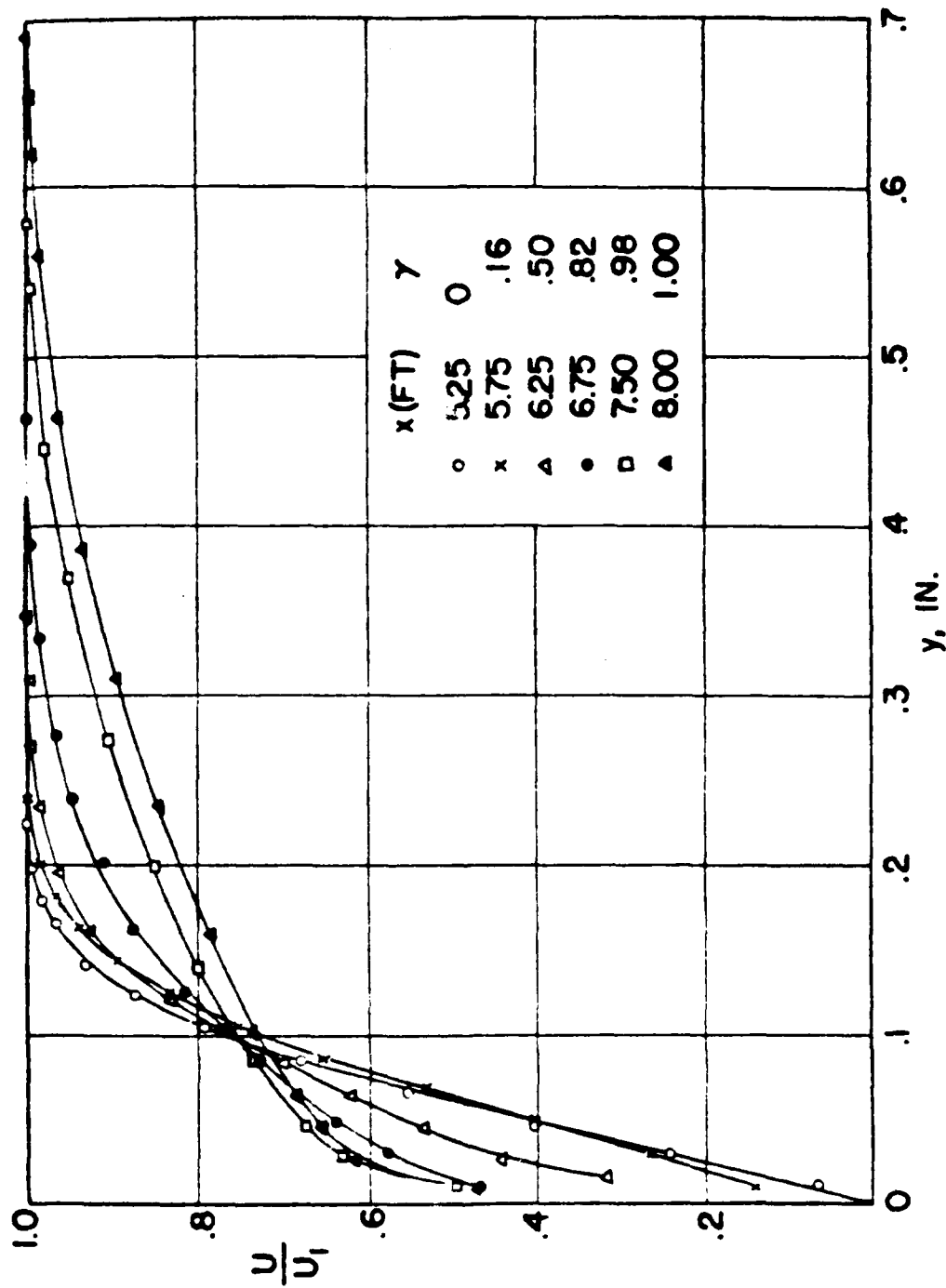
Further, decompose these streses as:

$$R_{ij} = \overline{R_{ij}} + \tilde{R}_{ij} + R_{ij}'$$

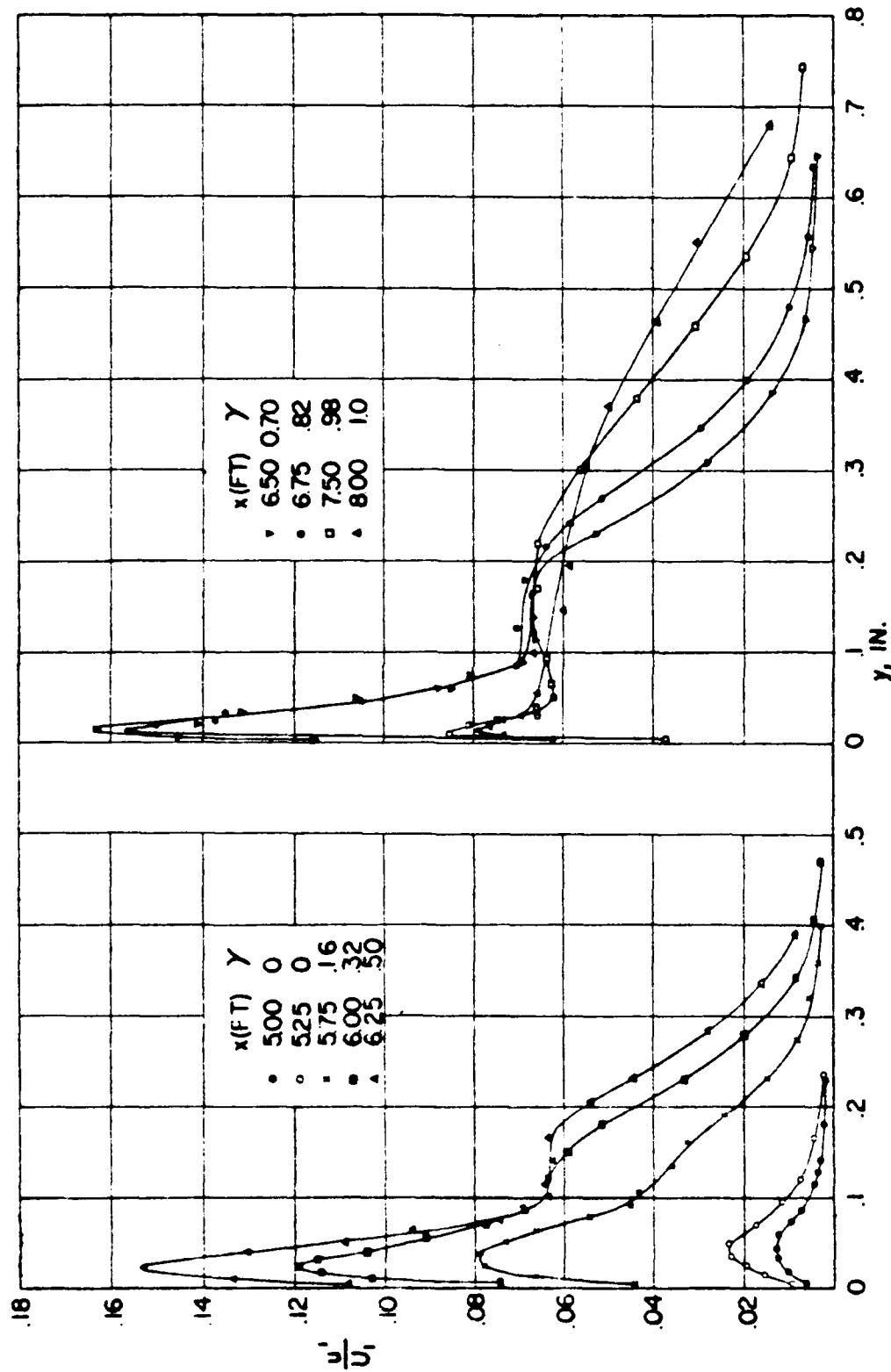
$$r_{ij} = \overline{r_{ij}} + \tilde{r}_{ij} + r_{ij}' .$$

ATTACHMENT 2

NBS REFERENCE DATA



Mean-velocity profiles through transition region.
 $U_1 = 80$ feet per second; free-stream turbulence, 0.03 percent.



Profiles of u'/U_1 through transition region. $U_1 = 80$ feet per second; free-stream turbulence, 0.03 percent.

END

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